

AP Calc AB Problem Set #3

Let p and q be real numbers and let f be the function defined by:

$$f(x) = \begin{cases} 1 + 2p(x-1) + (x-1)^2, & \text{for } x \leq 1 \\ qx + p, & \text{for } x > 1. \end{cases}$$

- (a) Find the value of q , in terms of p , for which f is continuous at $x = 1$.
- (b) Find the values of p and q for which f is differentiable at $x = 1$.
- (c) If p and q have the values determined in part (b), is f'' a continuous function? Justify your answer.

Let f be the function defined by $f(x) = x^4 - 3x^2 + 2$.

- (a) Find the zeros of f .
- (b) Write an equation of the line tangent to the graph of f at the point where $x = 1$.
- (c) Find the x -coordinate of each point at which the line tangent to the graph of f is parallel to the line $y = -2x + 4$.

Let f be the real-valued function defined by $f(x) = \sin^3 x + \sin^3 |x|$.

- (a) Find $f'(x)$ for $x > 0$.
- (b) Find $f'(x)$ for $x < 0$.
- (c) Determine whether $f(x)$ is continuous at $x = 0$. Justify your answer.
- (d) Determine whether the derivative of $f(x)$ exists at $x = 0$. Justify your answer.

Let f be the real-valued function defined by $f(x) = \sqrt{1+6x}$.

- (a) Give the domain and range of f .
- (b) Determine the slope of the line tangent to the graph of f at $x = 4$.
- (c) Determine the y -intercept of the line tangent to the graph of f at $x = 4$.
- (d) Give the coordinates of the point on the graph of f where the tangent line is parallel to $y = x + 12$.

Let f be the function defined by $f(x) = \frac{2x-5}{x^2-4}$.

- (a) Find the domain of f .
- (b) Write an equation for each vertical and each horizontal asymptote for the graph of f .
- (c) Find $f'(x)$.
- (d) Write an equation for the line tangent to the graph of f at the point $(0, f(0))$.

Let $f(x) = \sqrt{1 - \sin x}$.

- (a) What is the domain of f ?
- (b) Find $f'(x)$.
- (c) What is the domain of f' ?
- (d) Write an equation for the line tangent to the graph of f at $x = 0$.

Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$.

- (a) Find the domain of f .
- (b) Describe the symmetry, if any, of the graph of f .
- (c) Find $f'(x)$.
- (d) Find the slope of the line normal to the graph of f at $x = 5$.

A particle moves along the x -axis in such a way that its position at time t for $t \geq 0$ is given by $x = \frac{1}{3}t^3 - 3t^2 + 8t$.

- (a) Show that at time $t = 0$, the particle is moving to the right.
- (b) Find all values of t for which the particle is moving to the left.
- (c) What is the position of the particle at time $t = 3$?
- (d) When $t = 3$, what is the total distance the particle has traveled?

A particle starts at time $t = 0$ and moves on a number line so that its position at time t is given by $x(t) = (t - 2)^3(t - 6)$.

- (a) When is the particle moving to the right?
- (b) When is the particle at rest?
- (c) When does the particle change direction?
- (d) What is the farthest to the left of the origin that the particle moves?

A particle moves along a line so that at any time t its position is given by $x(t) = 2\pi t + \cos 2\pi t$.

- (a) Find the velocity at time t .
- (b) Find the acceleration at time t .
- (c) What are all values of t , $0 \leq t \leq 3$, for which the particle is at rest?
- (d) What is the maximum velocity?